Geometry 9

- This Slideshow was developed to accompany the textbook
  - *Larson Geometry*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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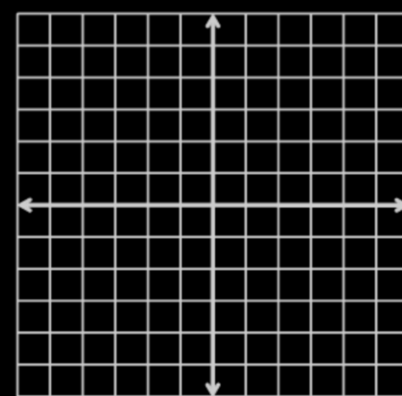
9.1 Translate Figures and Use Vectors

- Transformation
  - Moves or changes a figure
  - Original called preimage (i.e. ΔABC)
  - New called image (i.e. ΔA'B'C')

- Translation
  - Moves every point the same distance in the same direction

9.1 Translate Figures and Use Vectors

- Draw ΔRST with vertices R(2, 2), S(5, 2), and T(3, -2). Find the image of each vertex after the translation (x, y) → (x + 1, y + 2). Graph the image using prime notation.



R’(3, 4), S’(6, 4), T’(4, 0)

9.1 Translate Figures and Use Vectors

- The image of (x, y) → (x + 4, y – 7) is $\overline{P'Q'}$ with endpoints P'(-3, 4) and Q'(2, 1). Find the coordinates of the endpoints of the preimage.

P: x + 4 = -3 → x = -7
y – 7 = 4 → y = 11 P(-7, 11)
Q: x + 4 = 2 → x = -2
y – 7 = 1 → y = 8 Q(-2, 8)

9.1 Translate Figures and Use Vectors

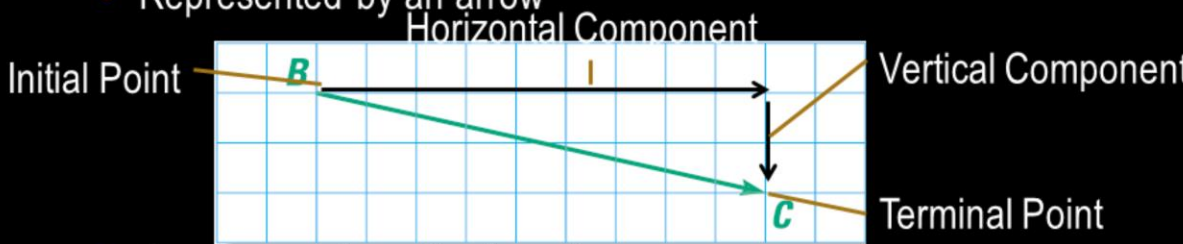
- Isometry
  - Transformation that preserves length and angle measure.
  - A congruence transformation

Translation Theorem
A translation is an isometry.

9.1 Translate Figures and Use Vectors

9.1 Translate Figures and Use Vectors

- Vector ($\overrightarrow{BC}$)
  - Measurement with Direction and Magnitude (size)
  - Represented by an arrow

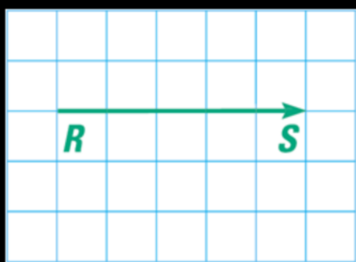


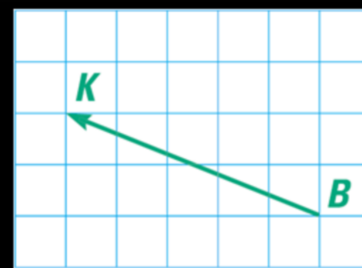
  - Component form ⟨hori, vert⟩
    - $\overrightarrow{BC} = \langle 9, -2 \rangle$

Can be used to describe translations

9.1 Translate Figures and Use Vectors

- Name the vector and write its component form





$$\overrightarrow{RS} = \langle 5,0 \rangle$$
$$\overrightarrow{TX} = \langle 0,3 \rangle$$
$$\overrightarrow{BK} = \langle -5,2 \rangle$$

9.1 Translate Figures and Use Vectors

- The vertices of ΔLMN are L(2, 2), M(5, 3), N(9, 1). Translate ΔLMN using vector $\langle -2,6 \rangle$.

- *576 #2-30 even, 34-40 even, 44, 48-54 even = 24*

Translation is (x, y)→(x-2, y+6)
L'(0, 8), M'(3, 9), N'(7, 7)

Answers and Quiz

- 9.1 Answers
- 9.1 Homework Quiz

9.2 Use Properties of Matrices

- Matrix
  - Rectangular arrangement of numbers in rows and columns
  - Each number is an element
  - $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ ← Row, ← Column
  - Dimension rows x columns
    - 2 x 3

9.2 Use Properties of Matrices

- Write a matrix to represent ΔABC with vertices A(3, 5), B(6, 7), C(7, 3).

- How many rows and columns are in a matrix for a hexagon?

x-coordinates go in first row; y-coordinates go in second row

$$\begin{bmatrix} 3 & 6 & 7 \\ 5 & 7 & 3 \end{bmatrix}$$

2 rows, 6 columns

9.2 Use Properties of Matrices

- Add and Subtract Matrices
  - Dimension must be equal
  - Add corresponding elements

- $\begin{bmatrix} 1 & -4 \\ 3 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix}$

$$\begin{bmatrix} -1 & -7 \\ -4 & -13 \end{bmatrix}$$

9.2 Use Properties of Matrices

- The matrix $\begin{bmatrix} 1 & 2 & 6 & 7 \\ 2 & -1 & 1 & 3 \end{bmatrix}$ represents quadrilateral JKLM. Write the translation matrix and the image matrix that represents the translation of JKLM 4 units right and 2 units down. Then find the coordinates of the image.

Translation matrix $\begin{bmatrix} 4 & 4 & 4 & 4 \\ -2 & -2 & -2 & -2 \end{bmatrix}$

Image coordinates $\begin{bmatrix} 1 & 2 & 6 & 7 \\ 2 & -1 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 4 & 4 & 4 \\ -2 & -2 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 10 & 11 \\ 0 & -3 & -1 & 1 \end{bmatrix}$

9.2 Use Properties of Matrices

- Matrix Multiplication
- Matrix multiplication can only happen if the number of columns of the first matrix is the same as the number of rows on the second matrix.
- You can multiply a 3x5 with a 5x2.
  - 3x**5** · **5**x2 → 3x2 will be the dimensions of the answer
- Because of this **order does matter**!

9.2 Use Properties of Matrices

$$\begin{bmatrix} 5 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 5 \cdot -3 + 1 \cdot -2 \end{bmatrix}$$

$$\begin{bmatrix} -17 \end{bmatrix}$$

9.2 Use Properties of Matrices

$$\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \cdot -2 + 2 \cdot 4 & 1 \cdot 1 + 2 \cdot 3 \\ 0 \cdot -2 + -3 \cdot 4 & 0 \cdot 1 + -3 \cdot 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 7 \\ -12 & -9 \end{bmatrix}$$

9.2 Use Properties of Matrices

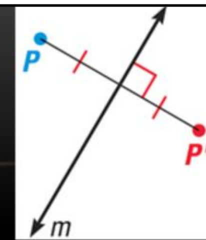
- *584 #2-32 even, 33, 38-44 even = 21*
- *Extra Credit 587 #2, 6 = +2*

Answers and Quiz

- 9.2 Answers

- 9.2 Homework Quiz

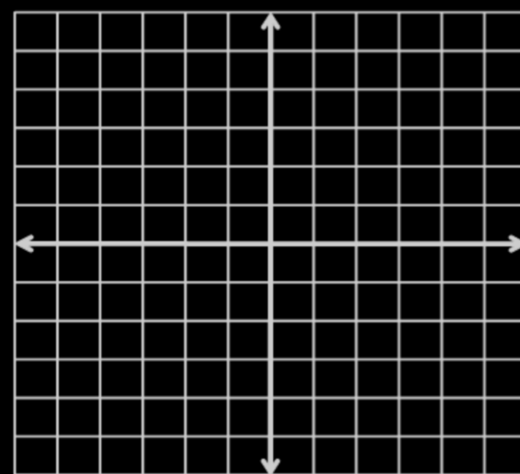
9.3 Perform Reflections



- Reflection
  - Transformation that uses a line like a mirror to reflect an image.
  - That line is called **Line of Reflection**
  - P and P' are the same distance from the line of reflection
  - The line connecting P and P' is perpendicular to the line of reflection

9.3 Perform Reflections

- Graph a reflection of ΔABC where A(1, 3), B(5, 2), and C(2, 1) in the line x = 2.



New points are A(3, 3), B(-1, 2), C(2, 1)

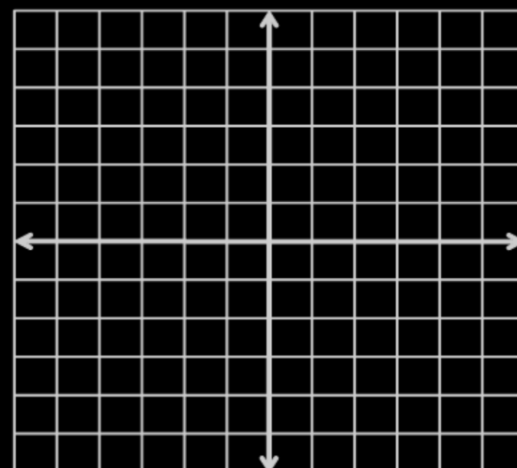
9.3 Perform Reflections

- Coordinate Rules for Reflections
  - Reflected in x-axis: (a, b) → (a, -b)
  - Reflected in y-axis: (a, b) → (-a, b)
  - Reflected in y = x: (a, b) → (b, a)
  - Reflected in y = -x: (a, b) → (-b, -a)

Reflection Theorem
A reflection is an isometry.

9.3 Perform Reflections

- Graph ΔABC with vertices A(1, 3), B(4, 4), and C(3, 1). Reflect ΔABC in the lines y = -x and y = x.



y = -x: new points A(-3, -1), B(-4, -4), C(-1, -3)
y = x: new points A(3, 1), B(4, 4), C(1, 3)

9.3 Perform Reflections

- Reflection Matrix
  - You can find the reflection of a polygon using matrix multiplication
  - Write the polygon vertices as a matrix
  - Multiply by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ for x-axis
    - Or $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ for y-axis
  - $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ for y = x
  - $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ for y = -x
  - [Reflection Matrix] [Polygon Matrix] = [Image Matrix]

Isometry: size and shape are unchanged

9.3 Perform Reflections

- The vertices of ΔLMN are L(-3, 3), M(1, 2), and N(-2, 1). Find the reflection of ΔLMN in the y-axis.

- *593 #4-24 even, 28, 40, 42-46 all = 18*

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 1 & -2 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

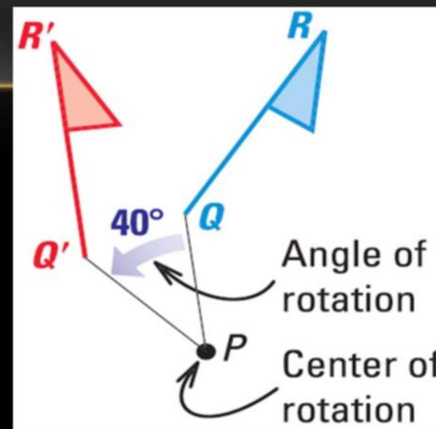
Answers and Quiz

- 9.3 Answers

- 9.3 Homework Quiz

9.4 Perform Rotations

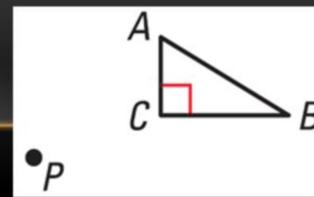
- Rotation
  - Figure is turned about a point called **center of rotation**
  - The amount of turning is **angle of rotation**



Rotation Theorem

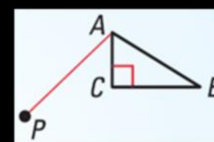
A rotation is an isometry.

9.4 Perform Rotations

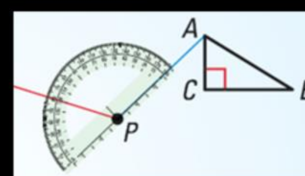


- Draw a 120° rotation of ΔABC about P.

1. Draw a segment from A to P.

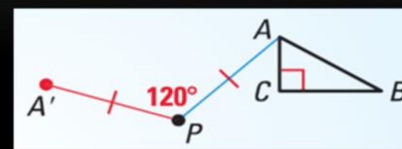


2. Draw a ray to form a 120° angle with $\overline{PA}$

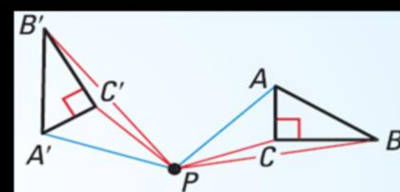


9.4 Perform Rotations

3. Draw A' so that PA' = PA

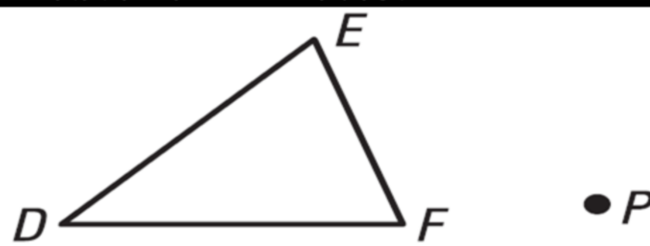


4. Repeat steps 1-3 for each vertex. Draw ΔA'B'C'.



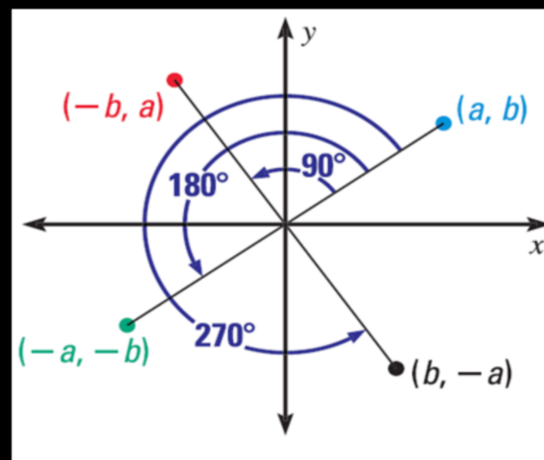
9.4 Perform Rotations

- Draw a 50° rotation of ΔDEF about P.



9.4 Perform Rotations

- Coordinate Rules for Counterclockwise Rotations about the Origin
- 90°: (a, b) → (-b, a)
- 180°: (a, b) → (-a, -b)
- 270°: (a, b) → (b, -a)



9.4 Perform Rotations

- Rotation Matrices (counterclockwise)
- 90°: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
- 180°: $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
- 270°: $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
- 360°: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- [Rotation Matrix] [Polygon Matrix] = [Image Matrix]

9.4 Perform Rotations

- If E(-3, 2), F(-3, 4), G(1, 4), and H(2, 2). Find the image matrix for a 270° rotation about the origin.

- *602 #4-28 even, 32, 36-40 even, 41-46 all = 23*

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -3 & -3 & 1 & 2 \\ 2 & 4 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 4 & 2 \\ 3 & 3 & -1 & -2 \end{bmatrix}$$

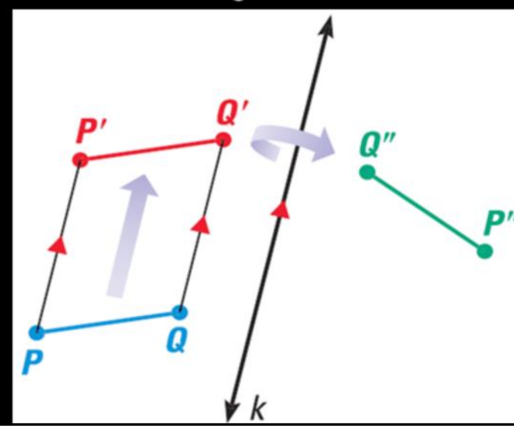
ANSWERS AND QUIZ

- 9.4 Answers

- 9.4 Homework Quiz

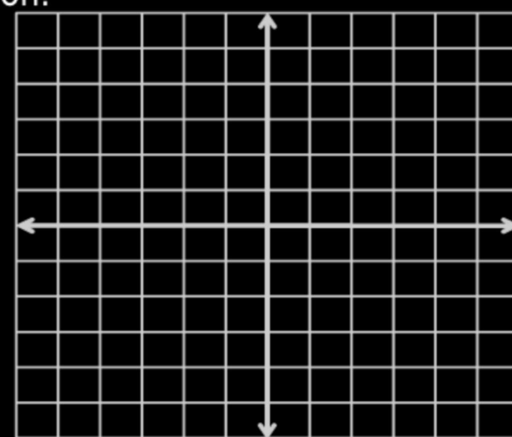
9.5 Apply Compositions of Transformations

- Composition of Transformations
  - Two or more transformations combined into a single transformation
- Glide Reflection
  - Translation followed by Reflection



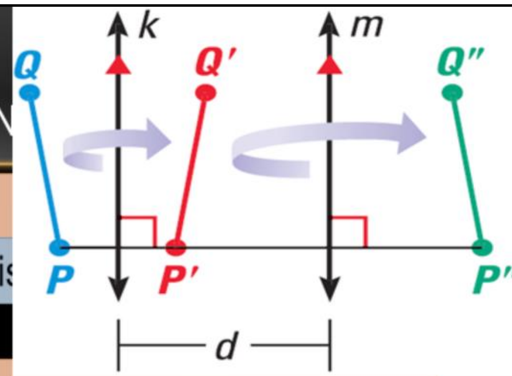
9.5 Apply Compositions of Transformations

- The vertices of ΔABC are A(3, 2), B(-1, 3), and C(1, 1). Find the image of ΔABC after the glide reflection.
  - Translation: (x, y) → (x, y – 4)
  - Reflection: Over y-axis



New points after translation: A’(3, -2), B’(-1, -1), C’(1, -3)
New points after reflection: A’’(-3, -2), B’’(1, -1), C’’(-1, -3)

9.5 Apply Compositions of Tran



Composition Theorem

A composition of two (or more) isometries is

Reflections in Parallel Lines Theorem

If lines *k* and *m* are parallel, then a reflection in line *k* followed by a reflection in line *m* is the same as a translation.

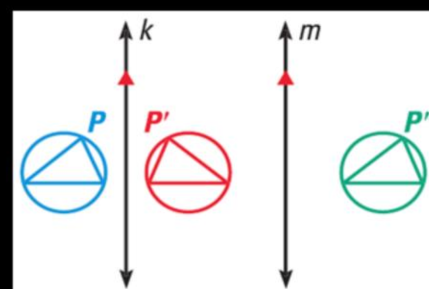
If P'' is the image of P, then

1. $\overline{PP''}$ is ⊥ to *k* and *m*, and
2. *PP*" = 2*d* where *d* is the distance between *k* and *m*

9.5 Apply Compositions of Transformations

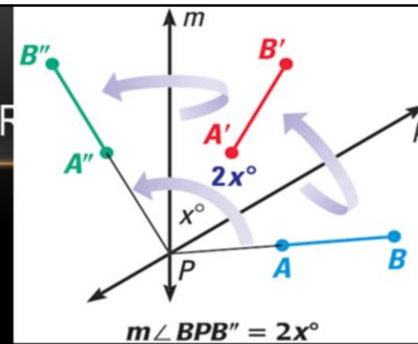
Use the figure below. The distance between line *k* and *m* is 1.6 cm.

1. The preimage is reflected in line *k*, then in line *m*. Describe a single transformation that maps the blue figure to the green.
2. What is the distance from P and P''?



1. Translation in the x direction
2. 2(1.6 cm) = 3.2 cm (Reflections in Parallel Lines Thrm)

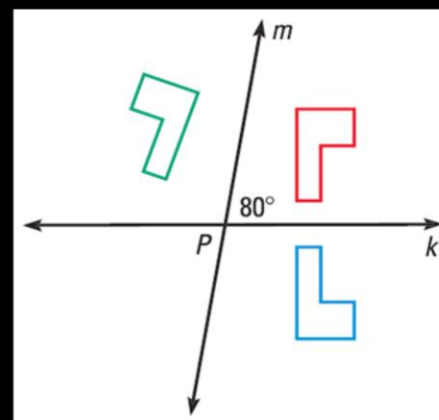
9.5 Apply Compositions of Transfor



Reflections in Intersecting Lines Theorem
If lines *k* and *m* intersect at point P, then a reflection in line *k* followed by a reflection in line *m* is the same as a rotation about point P.
The angle of rotation is 2x°, where x° is the measure of the acute or right angle formed *k* and *m.*

9.5 Apply Compositions of Transformations

- In the diagram, the preimage is reflected in line *k*, then in line *m*. Describe a single transformation that maps the blue figure to the green.



- *611 #2-30 even, 40-48 even = 20*
- *Extra Credit 615 #2, 8 = +2*

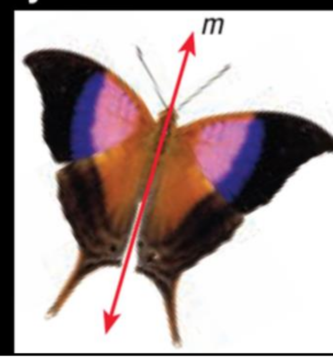
Counterclockwise rotation of 160° about point P

Answers and Quiz

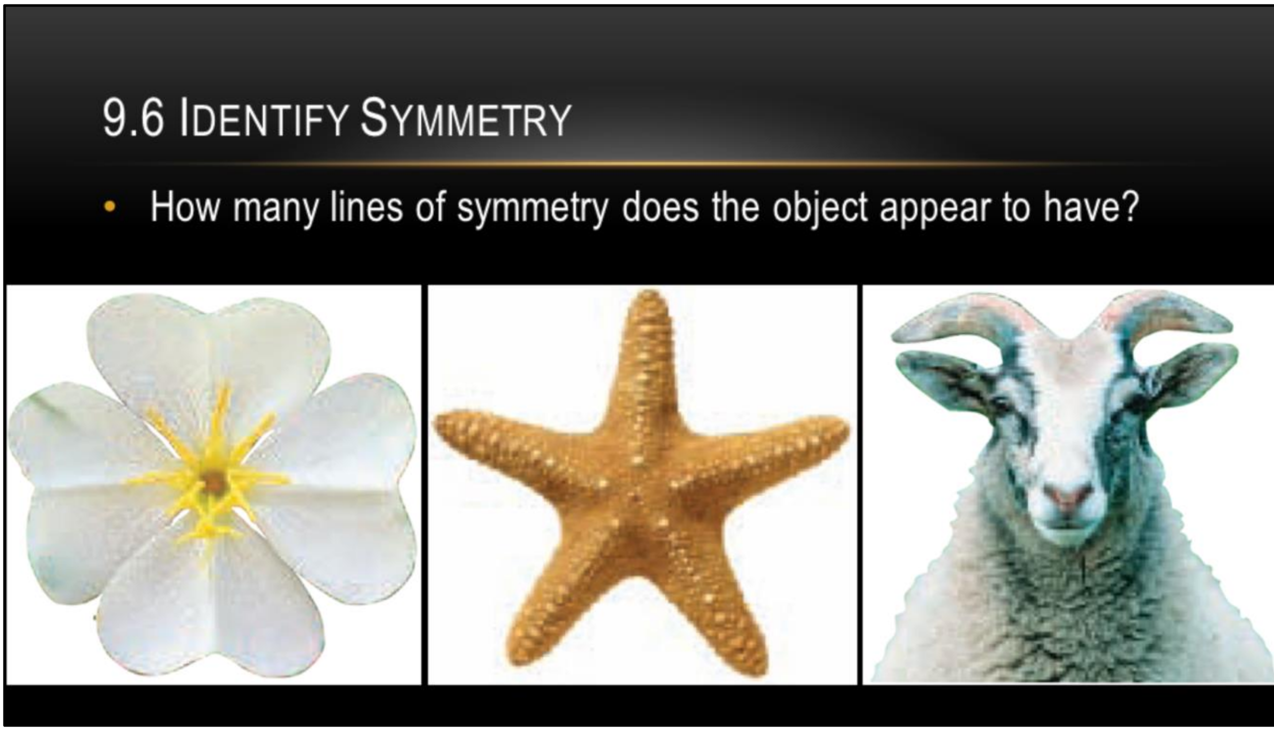
- 9.5 Answers
- 9.5 Homework Quiz

9.6 Identify Symmetry

- Line symmetry
  - The figure can be mapped to itself by a reflection
  - The line of reflection is called **Line of Symmetry**



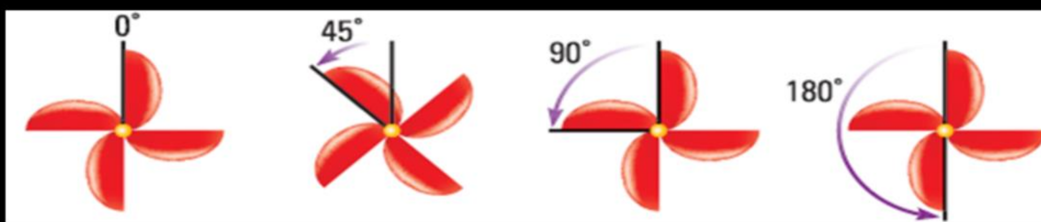
- Humans tend to think that symmetry is beautiful



Flower: 4 lines of symmetry
Sea Star: 5 lines of symmetry
Goat: 1 line of symmetry

9.6 Identify Symmetry

- Rotational Symmetry
  - The figure can be mapped to itself by a rotation of 180° or less about the center of the figure
  - The center of rotation is called the **Center of Symmetry**



Note: the 45° is not a symmetry

9.6 Identify Symmetry

- Does the figure have rotational symmetry? What angles?



- *621 #4-20 even, 24-34 even, 37-45 all = 24*

Rhombus: 180°
Octagon: 90°, 180°
Triangle: none

Answers and Quiz

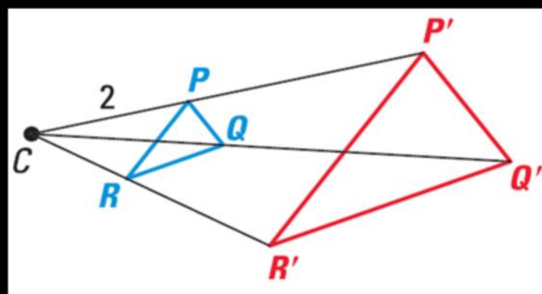
- 9.6 Answers
- 9.6 Homework Quiz

9.7 Identify and Perform Dilations

- Dilation
  - Enlarge or reduce
  - Image is similar to preimage
  - Scale factor is k
    - If $0 < k < 1$, then reduction
    - If $k > 1$, then enlargement

9.7 Identify and Perform Dilations

- The image point P' lies on $\overrightarrow{CP}$. The scale factor *k* is a positive number such that $k = \frac{CP'}{CP}$ and $k \neq 1$



- Scale factor is $\frac{\text{image}}{\text{preimage}}$

Scale

9.7 Identify and Perform Dilations

- Draw and label ΔRST, then construct a dilation of ΔRST with R as the center of dilation and a scale factor of 3.

1. Draw ΔRST, then draw rays $\overrightarrow{RS}$ and $\overrightarrow{RT}$
2. Using a ruler, measure RS. Multiply by the scale factor. Using the ruler mark this length RS' on $\overrightarrow{RS}$. Repeat for the other rays.
3. Draw ΔR'S'T'

9.7 Identify and Perform Dilations

- Scalar matrix multiplication
  - When you multiply a number by matrix, distribute to each element.
- Simplify
- $5 \cdot \begin{bmatrix} 2 & 1 & -10 \\ 3 & -4 & 7 \end{bmatrix}$

$$\begin{bmatrix} 10 & 5 & -50 \\ 15 & -20 & 35 \end{bmatrix}$$

9.7 Identify and Perform Dilations

- Dilation using matrices (center at origin)
  - Scale factor [polygon matrix] = [image matrix]

- The vertices of ΔRST are R(1, 2), S(2, 1), and T(2, 2). Use scalar multiplication to find the vertices of ΔR'S'T' after a dilation with its center at the origin and a scale factor of 2.

- *629 #2-28 even, 32-36 even, 40, 43-49 all = 25*
- *Extra Credit 632 #2, 6 = +2*

$$2\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}2 & 4 & 4\\ 4 & 2 & 4\end{bmatrix}$$

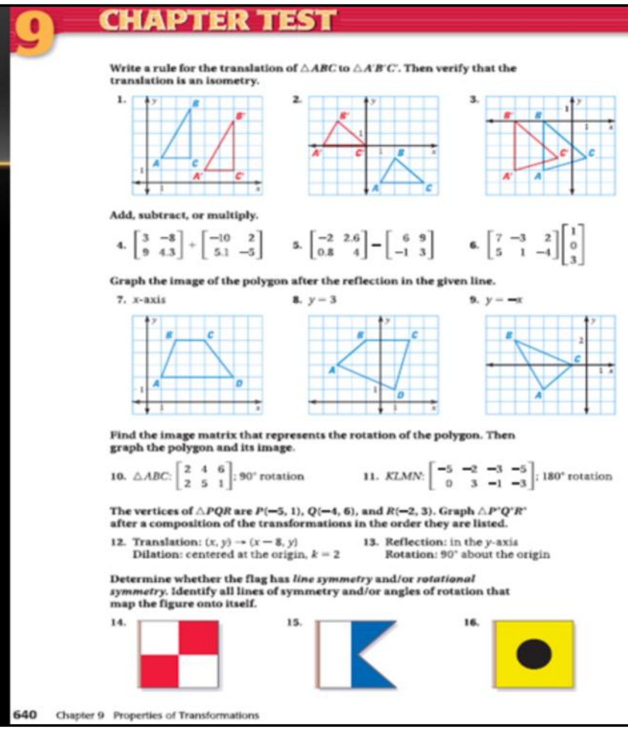
Answers and Quiz

- 9.7 Answers

- 9.7 Homework Quiz

9.Review

- *640 #1-16 all = 16*



9 CHAPTER TEST

Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$. Then verify that the translation is an isometry.

1.
2.
3.

Add, subtract, or multiply.

4. $\begin{bmatrix} 3 & -8 \\ 9 & 4.3 \end{bmatrix} + \begin{bmatrix} -10 & 2 \\ 5.1 & -5 \end{bmatrix}$

5. $\begin{bmatrix} -2 & 2.6 \\ 0.8 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ -1 & 3 \end{bmatrix}$

6. $\begin{bmatrix} 7 & -3 & 2 \\ 5 & 1 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

Graph the image of the polygon after the reflection in the given line.

7. x-axis
8. $y = 3$
9. $y = -x$

Find the image matrix that represents the rotation of the polygon. Then graph the polygon and its image.

10. $\triangle ABC$: $\begin{bmatrix} 2 & 4 & 6 \\ 2 & 5 & 1 \end{bmatrix}$; 90° rotation

11. $KLMN$: $\begin{bmatrix} -5 & -2 & -3 & -5 \\ 0 & 3 & -1 & -3 \end{bmatrix}$; 180° rotation

The vertices of $\triangle PQR$ are $P(-5, 1)$, $Q(-4, 6)$, and $R(-2, 3)$. Graph $\triangle P''Q''R''$ after a composition of the transformations in the order they are listed.

12. **Translation:** $(x, y) \rightarrow (x - 8, y)$
Dilation: centered at the origin, $k = 2$

13. **Reflection:** in the y-axis
Rotation: 90° about the origin

Determine whether the flag has *line symmetry* and/or *rotational symmetry*. Identify all lines of symmetry and/or angles of rotation that map the figure onto itself.

14.
15.
16.

640 Chapter 9 Properties of Transformations